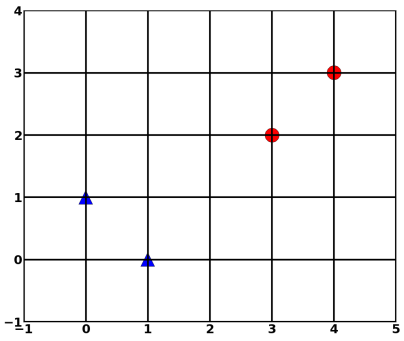
CS 189 Introduction to Machine Learning
Summer 2018 DIS11

1 Support Vector Machines

Assume we are given dataset $\mathcal{D} = \{(\mathbf{x}_i, y_i)\}_{i=1}^n$, where $x_i \in \mathbb{R}^d$ and $y_i \in \{+1, -1\}$.

In SVM the goal is to find some hyperplane which separates the positive from the negative examples, such that the margin (the minimum distance from the decision boundary to the training points) is maximized. Let the equation for the hyperplane be $\mathbf{w}^\top \mathbf{x} + b = 0$.

(a) You are presented with the following set of data (triangle = +1, circle = -1):



Find the equation (by hand) of the hyperplane $\mathbf{w}^\top \mathbf{x} + b = 0$ that would be used by an SVM classifier. Which points are support vectors?

(b) We typically frame an SVM problem as trying to *maximize* the margin. Explain intuitively why a bigger margin will result in a model that will generalize better, or perform better in practice.

(c) Show that the width of an SVM slab with linearly separable data is $\frac{2}{\|\mathbf{w}\|}$.

(d) Write SVM as an optimization problem. Conclude that maximizing the margin is equivalent to minimizing $\|\mathbf{w}\|$.

(e) Will moving points which are not support vectors further away from the decision boundary effect the SVM's hinge loss?

2 Curse of Dimensionality

We have a training set: $(\mathbf{x}^{(1)}, y^{(1)}), \ldots, (\mathbf{x}^{(n)}, y^{(n)})$, where $\mathbf{x}^{(i)} \in \mathbb{R}^d$. To classify a new point $\mathbf{x}$, we can use the nearest neighbor classifier:

$$\text{class}(\mathbf{x}) = y^{(i^*)} \quad \text{where } \mathbf{x}^{(i^*)} \text{ is the nearest neighbor of } \mathbf{x}.$$

Assume any data point $\mathbf{x}$ that we may pick to classify is inside the Euclidean ball of radius 1, i.e. $\|\mathbf{x}\|_2 \leq 1$. To be confident in our prediction, in addition to choosing the class of the nearest neighbor, we want the distance between $\mathbf{x}$ and its nearest neighbor to be small, within some positive ϵ:

$$\|\mathbf{x} - \mathbf{x}^{(i^*)}\|_2 \leq \epsilon \quad \text{for all } \|\mathbf{x}\|_2 \leq 1. \tag{1}$$

What is the minimum number of training points we need for inequality (1) to hold (assuming the training points are well spread)? How does this lower bound depend on the dimension d?

Hint: Think about the volumes of the hyperspheres in d dimensions.